In the Specification

Please amend the paragraph that appears on page 4, line 15 through page 5, line 21 as follows:

Figure 1 is a block diagram of a switched beam beamforming system in accordance with the prior art. An antenna array 12 ~~comprising~~ comprises multiple antennas 12_1, 12_2,..., 12_N that are arranged at a receiving station 10, e.g., at the top of a cellular telephone system cell tower. The outputs of those antennas are fed into an analog NxB beamforming circuit 14, wherein N is the number of antennas and B is the number of beams generated by the beamforming circuitry from the antenna signals. The analog beamforming circuitry 14 weighs and combines the RF antenna output signals on lines 13_1, 13_2,..., 13_N in accordance with a scheme dictated by beamforming control circuit 121 to produce the beam signals 15_1, 15_2,..., 15_B, each beam focused on an angular portion of the reception area. Beamforming circuit 121 may be a DSP executing a predetermined algorithm. The number of beams, B, typically is less than or equal to the number of antennas, N, in the antenna array. Each beam signal 15_1, 15_2,..., 15_B is passed through frequency down converting circuitry 16_1, 16_2,..., 16_B for converting the beam signals from the RF frequency range to the baseband range. Each of those signals is digitized by an analog to digital (A/D) converter 18_1, 18_2,..., 18_B. The outputs of the A/D converters are then each input to a fading multipath and multi-user channel estimation circuit, 20_1, 20_2,..., 20_B. Each of those circuits generates L output signals for each of M simultaneous transmitters at use in the given geographic

area, where L is the number of paths per transmitter that the circuitry is designed to process simultaneously (e.g., typically around 3 or 4) and M is the number of simultaneous transmitters using the receive station. Those outputs are input to a minimum variance selector 22 for all of the paths and users. The minimum variance selector generates a path estimate and a path estimate error for each path of each user, i.e., L x M path estimates and L x M path estimate errors.

Please amend the paragraph that appears on page 6, lines 7 through 14 as follows:

In the analog N x B beamforming circuitry 14, for each beam, dedicated RF combining circuitry is necessary, i.e., there are ~~N~~ B copies of essentially identical circuitry for processing the N ~~antenna signals~~ beams on lines ~~13~~ 15_1, ~~13~~ 15_2,..., ~~13~~ 15_B. RF band hardware is much more expensive than DSPs and other baseband circuitry. Also, as shown in Figure 1, each beam requires a dedicated frequency down converting circuit 16 and a dedicated analog-to-digital converter 18.

Please amend the paragraph that appears on page 11, line 19 through page 12, line 19 as follows:

The N vector channel impulse response from the *m*th user to the antenna array is given by

$$h_m(t,\tau)=\sum_{l=1}^{L_m(t)}\alpha(\phi_{m,l})\rho_{m,l},l(t)e^{j\psi m,l(t)}\delta(t-\tau_{m,l}(t)) \tag{4}$$

where $L_m(t)$ is the known - possibly time varying - number of resolvable paths. $\delta(t)$ is the Dirac delta function. ~~$\phi_{M,1}(t)$~~ $\phi_{M,1}$ is the known direction of arrival (DOA) or angle of arrival (AOA) of the mth user's signal via the lth path. The AOA is measured anti-clockwise from positive real axis. $\rho_{m,1}(t)$, ~~$\psi_{m,1}(t)$~~ $\psi_{m,,1}(t)$ and $T_{k,1}(t)$ are the signal attenuation, phase shift and time delay of the lth multipath component for user m, respectively, $\alpha(\phi)$ is known as the steering vector, and denotes the response of the antenna array to a signal impinging onto the array from an angle ϕ. For a uniform linear array, the steering vector is given by

$$\alpha(\phi)=[g_1(\phi)\alpha_1(\phi),g_2(\phi)\alpha_2(\phi),\ldots,g_N(\phi)\alpha_N(\phi)] \tag{5}$$

where $g_n(\phi)$ denotes the directivity (gain) of the *n*th antenna element impinging on the array from angle ϕ. Here we assume for illustrative purposes that all the elements have unit directivity, i.e., $g_1(\phi)=\ldots=g_N(\phi)=1$. For our uniform linear array, we have

$$\alpha_n(\phi)=e^{j\pi(n-1)\cos(\phi)},\forall n\in\{1,\ldots,N\} \tag{6}$$

Please amend the paragraph that appears on page 14, line 21 through page 15, line 7 as follows:

Independent Rayleigh distributed multiplicative channel coefficients: We assume the channel coefficients $\alpha_{m,1}(t)$ in Eq. (8) between symbol intervals are modeled as

independent zero-mean circular white Gaussian noise processes and remain constant during each symbol period. In particular, for $n, n' = -\infty, \ldots, \infty$, $m, m' \in \{1, \ldots, M\}$ and $l, l' \in \{1, \ldots, L\}$~~.~~,

$$\alpha_{m,l}(nT_b+\Delta) \sim N(0, \sigma_{m,l}^2) \quad (12)$$

$$\alpha_{m,l}(nT_b+\Delta) = \alpha_{m,l}(nT_b+T_c+\Delta) = \cdots \alpha_{m,l}(nT_b+(G-1)T_c+\Delta) \quad (13)$$

$$V_{k,m,l} = \begin{cases} \chi_{k,m,l,} & \forall k \in \{\ldots,-2(G-1),-G-1),0,G-1,2(G-1),\ldots\} \\ 0. & otherwise \end{cases}$$

$$E\{\alpha_{m,l}(nT_b+\Delta)\alpha_{m',l'}(n'T_b+\Delta)\} = \delta_{n,n'}\delta_{m,m'}\delta_{l,l'}\delta^2_{m,l} \quad (14)$$

where $\delta_{k,k'}$ denotes the Kronecker delta function, that is $\delta_{k,k'}=1$, if $k=k'$, and $\delta_{k,k'}=0$ otherwise.

Please amend the paragraph on page 15, lines 15 through 20 as follows:

Discrete-time state-space model: For notational convenience, we rewrite the kth sampled array output after beamforming, given by Eq. (10), as follows

$$Y_k = f_k^H \tau_k + f_k^H W_k \quad (16)$$

where $y_k \triangleq y(kT_c+\Delta)$, $f_k \triangleq f(kT_c+\Delta)$ and $w_k \triangleq w(kT_c+\Delta)$. Here we use the subscript _k_ to denote the kth sample.

Please amend the paragraph beginning on page 15, line 21 through page 18, line 4 as follows:

Let ~~$\chi_{k,m,l}$~~ $\chi_{k,m,l}$ denote the attenuated received spectrum signal at time k

for the *m*th user via the lth path, i.e.

$$\chi_{k,m,l} \triangleq b^{c}_{k,m} c_{k,m} \alpha_{k,m,l} \qquad (17)$$

where $\alpha_{k,m,l} \triangleq \alpha_{m,k}(kT_c+\Delta)$. Let

$$V_{k,m,l} = \begin{cases} \chi_{k,m,l}, & \forall k \in \{\ldots,-2(G-1),-(G-1),0,G-1,2(G-1),\ldots\} \\ 0. & otherwise \end{cases} \qquad (18)$$

denote the realization of the attenuated and phase shifted spread spectrum $\chi_{k,m,l}$ every G samples. Since the product $b^{c}_{k,m} c_{k,m}$ yields ± 1, and from our statistical assumptions on the channel fading, $V_{k,m,l}$ is complex white Gaussian distributed random variable, such that

$$V_{k.m.l} \sim N(0,\sigma^2_{m,l}), \text{ for } k \in \{\ldots,-2(G-1), -(G-1),0,G-1,2(G-1),\ldots\} \qquad (19)$$

and $\cancel{U_{k,m,l}}$ $\underline{V}_{k,m,l} = 0$, otherwise. Let

$$h_k = \begin{cases} 0, \forall k \in \{\ldots,-2(G-1),-(G-1),0,G-1,2(G-1),\ldots\} \\ 1, otherwise \end{cases} \qquad (20)$$

The attenuated phase shifted received spread spectrum signal $\chi_{k,m,l}$ can be written in the following scalar recursive form

$$\chi_{k,m,l} = h_k c_{k,m} c_{k-1,m,l} \chi_{k-1,m,l} + V_{k,m,l} \qquad (21)$$

We group the received signals $\chi_{k,m,l}$ at time index k in a $ML\times 1$ vector χ_k as follows

$$\cancel{\chi^H_k \triangleq (\chi_{k,1,1},\ldots \chi_{k,1,L},\ldots,\chi_{k,M,1},\ldots,\chi_{k,M,L})}$$

$$\underline{\chi^H_k \triangleq (\chi_{k,1,1}, \ldots, \chi_{k,1,L}, \ldots, \chi_{k,M,1}, \ldots, \chi_{k,M,L})^H} \qquad (22)$$

If we define $V_k^H \triangleq (V_{k,1,1}, \ldots, V_{k,1,L}, \ldots, V_{k,M,1}, \ldots, V_{k,M,L})^H$, then we have the following vector state equation for the time evolution of the received spread spectrum signals for all user and all paths

$$\chi_k^H \triangleq \; = A_k \chi_{k-1} + V_k \tag{23}$$

where

$$V_k \sim N(0_{MLx1}, Q), \forall \; k \in \{\ldots, -2(G-1), -(G-1), 0, G-1, 2(G-1), \ldots\} \tag{24}$$

$$V_k + 0, \text{ otherwise} \tag{25}$$

and

$$A_k = \mathrm{diag}\left(\underbrace{h_k c_{k-1,1} c_{k,1} \cdots, h_k c_{k-1,1} c_{k,1}}_{L\,Times}, \ldots, \underbrace{h_k c_{k-1,M} c_{k,M}, \ldots, h_k c_{k-1,M} c_{k,M}}_{L\,Times}\right) \tag{26}$$

~~$Q = diag(\sigma_{1,1}^2, \ldots, \sigma_{1}^2, \ldots, \sigma_{M,1}^2, \ldots, \sigma_{M,L}^2)$~~ $\underline{Q = diag(\sigma_{1,1}^2, \ldots, \sigma_{1}^2, \ldots, \sigma_{M,1}^2, \ldots, \sigma_{M,L}^2)}$

(27)

where A_k and Q are MLxML diagonal matrices.

Please amend the paragraph beginning on page 17, line 14 through page 18, line 10 as follows:

Let ϕ denote the vector of all angle of arrivals for all M users.

$$\phi^H = (\phi_{1,1}, \ldots, \phi_{1,L}, \ldots, \phi_{M,1}, \ldots, \phi_{M,L})^H \tag{28}$$

We use $A(\phi)$ to denote the N x ML matrix response of the array, defined as follows

$$A(\phi) = (a(\phi_{1,1}), \ldots, a(\phi_{1,L}), \ldots, a(\phi_{M,1}), \ldots, a(\phi_{M,L})^H \tag{29}$$

The ~~((m-1)xL=1) the~~ <u>((m-1)xL=1)th</u> column vector in $A(\phi)$ corresponds to the

antenna array vector response of a signal impinging onto the antenna array from angle $\phi_{m,l}$.

Please amend the paragraph appearing on page 18, lines 5 through 10 as follows:

Using Eqs. (23) and (29), the observation Eq. (16) is equivalently written as

$$y_k = f_k^H A(\phi)\chi_k + f_k^H W_k \tag{30}$$

Equations (23) and (30) ~~from~~ form the discrete-time, state-space version of the DS-CDMA beamforming linear antenna array system.

Please amend the paragraph that appears on page 18, line 11 through page 19, line 6 as follows:

1. **Optimal Chip-Rate Switched-Beam Design**

In this subsection, we formulate the estimation objectives. The aim is to design the time varying switched-beam beamforming vector f_k, such that the minimum estimation error in estimating the received spread spectrum signal χ_k defined in Eq. (22) is obtained. We ~~chose~~ choose the beamforming vector f_k to be a function of the data from time index 0 up to time index k-1, Furthermore, f_k belongs to a set of fixed beam array patterns F. Thus,

$$f_k = f_k(Y_0, Y_1, \ldots, Y_{k-1}) \in F \tag{31}$$

for k = 1,2,.... The aim is to select the class of beam patterns F and optimally select the sequence of beam-patterns from the set F, such that the estimation errors in estimating the spread spectrum signals are minimized. We chose the following optimization function

$$h_k = E\{(\chi_k - \chi_{k/k})^H J(\chi_k - \chi_{k/k})\} \tag{32}$$

where $\chi_{k/k} \triangleq E\{\chi_k/Y_k\}$ is the conditional mean estimate of χ_k, conditioned on the data ~~$Y_k \triangleq (y_0, Y1, ..., Yk)$~~ $Y_k \triangleq (y_0, Y1, ..., Yk)$, and J is some predetermined – user defined - weighting matrix. We choose the sequence of optimal beamforming vector ~~$\hat{f}_0, \hat{f}_1, \hat{f}_2, \ldots),$~~ $(\hat{f}_0, \hat{f}_1, \hat{f}_2, \ldots f_k),$ as follows

$$(\hat{f}_0, \hat{f}_1, ..., \hat{f}_k) = \underset{f_0, f_1, \ldots, f_k}{\arg\max}\, E\{(\chi_k - \chi_{k/k})^H J(\chi_k - \chi_{k/k})\} \tag{33}$$

subject to Eq. (31).

Please amend the paragraph that appears on page 20, lines 10 through 12 as follows:

From the previous discussion, we ~~assumed~~ assume M x L steering vectors associated with M x L known angle-of-arrivals $\phi_{m,1}$. The set of conventional beamforming array vectors ~~are~~ is given by

$$F_2 = \{\alpha(\phi_{1,1}), \ldots, \alpha(\phi_{1,L}), \ldots, \alpha(\phi_{M,1}), \ldots, \alpha(\phi_{M,L})\} \tag{35}$$

Please amend the paragraph that appears beginning on page 20, line 14 through page 21, line 8 as follows:

Optimal Beamforming: The optimal beamforming vector is derived by minimizing the average output power of the beamforming array, while ensuring the unity response in the desired direction s_0. Assuming $f = f_k$ for all k, we solve the following optimization problem

$$\min_f E\{yy^H\}, subject\ to\ f^H s_0 = 1 \tag{36}$$

Using Lagrange's multiplier method, the optimal beamformer *f* can be expressed as follows

$$f = \frac{D^{-1}s_0}{s_0^H D^{-1} s_0} \tag{37}$$

where D denotes the average signal, multi-user interference (MUI) plus noise ~~power~~ covariance.

From our statistical assumptions on our signal model, we can show the following result

$$D = A(\phi)QA^H(\phi) + R \tag{38}$$

Note that D is a positive definite matrix since Q and R are positive definite matrices. Thus, the set of optimal beamforming steering vectors are ~~give~~ given by

$$F_3 = \left\{ \frac{D^{-1}\alpha(\phi_{1,1})}{\alpha^H(\phi_{1,1})D^{-1}\alpha(\phi_{1,1})}, \ldots, \frac{D^{-1}\alpha(\phi_{1,L})}{\alpha^H(\phi_{1,L})D^{-1}\alpha(\phi_{1,L})}, \ldots, \frac{D^{-1}\alpha(\phi_{M,1})}{\alpha^H(\phi_{M,1})D^{-1}\alpha(\phi_{M,1})}, \ldots, \frac{D^{-1}\alpha(\phi_{M,L})}{\alpha^H(\phi_{M,L})D^{-1}\alpha(\phi_{M,L})} \right\}$$